

Combining Ratio Estimation for Low Density Parity Check (LDPC) Coding

C. Saad Mahmoud, Jianjun (David) Ni

Abstract

The Low Density Parity Check (LDPC) Code decoding algorithm make use of a scaled receive signal derived from maximizing the log-likelihood ratio of the received signal. The scaling factor (often called the combining ratio) in an AWGN channel is a ratio between signal amplitude and noise variance. Accurately estimating this ratio has shown as much as 0.6 dB decoding performance gain. This presentation briefly describes three methods for estimating the combining ratio: a ‘Pilot-Guided’ estimation method, a ‘Blind’ estimation method, and a Simulation-Based Look-Up table. The ‘Pilot Guided’ Estimation method has shown that the maximum likelihood estimates of signal amplitude is the mean inner product of the received sequence and the known sequence, the attached synchronization marker (ASM), and signal variance is the difference of the mean of the squared received sequence and the square of the signal amplitude. This method has the advantage of simplicity at the expense of latency since several frames worth of ASMs. The ‘Blind’ estimation method’s maximum likelihood estimator is the average of the product of the received signal with the hyperbolic tangent of the product combining ratio and the received signal. The root of this equation can be determined by an iterative binary search between 0 and 1 after normalizing the received sequence. This method has the benefit of requiring one frame of data to estimate the combining ratio which is good for faster changing channels compared to the previous method, however it is computationally expensive. The final method uses a look-up table based on prior simulated results to determine signal amplitude and noise variance. In this method the received mean signal strength is controlled to a constant soft decision value. The magnitude of the deviation is averaged over a predetermined number of samples. This value is referenced in a look up table to determine the combining ratio that prior simulation associated with the average magnitude of the deviation. This method is more complicated than the ‘Pilot-Guided’ Method due to the gain control circuitry, but does not have the real-time computation complexity of the ‘Blind’ Estimation method. Each of these methods can be used to provide an accurate estimation of the combining ratio, and the final selection of the estimation method depends on other design constraints.



Combining Ratio Estimation for Low Density Parity Check (LDPC) Coding

Saad Mahmoud/ESCG

30 March 2009



Outline

- **Background & Purpose**
- **Method 1: The Pilot-Guided Estimation Method**
- **Method 2: The Blind Estimation Method**
- **Method 3: Prior Simulation – Look-up Table**
- **Summary**
- **References**
- **Extra Slides**



Background & Purpose

- The LDPC decoder needs a scaled received signal as its input
- Scaling Factor used is derived from the log likelihood ratio of the received signal

$$\frac{2A}{\sigma^2} y$$

- Implies a scaling factor of 2α where $\alpha = \frac{A}{\sigma^2}$
- Though referred to as SNR, it is a combining ratio. In digital communications, symbol SNR is:
$$\frac{E_s}{N_o} = \frac{A^2}{2\sigma^2}$$
- Without a good estimate there is as much as 0.6dB decoding performance loss associated with the LDPC decoding algorithm.

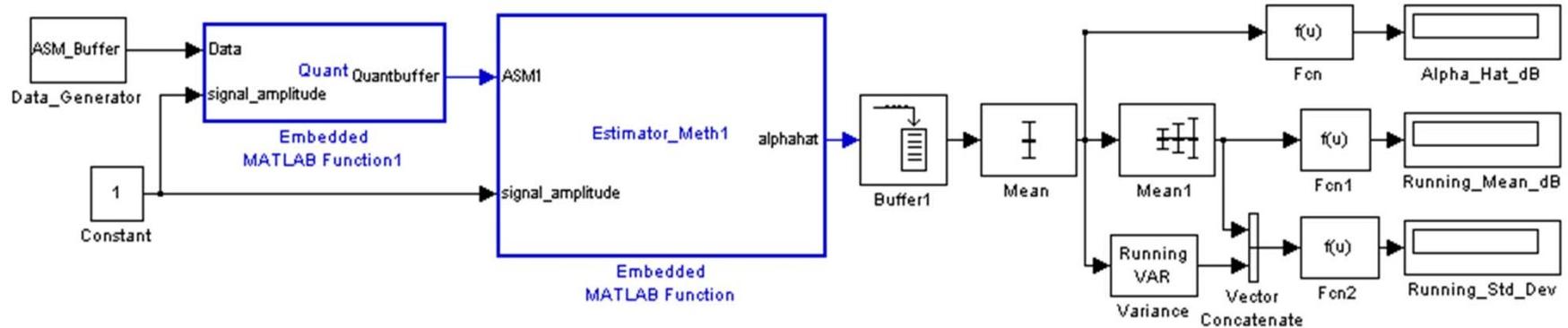


Purpose

- **To compare methods of determining the combining ratio**
 - Method 1: The Pilot Guided Estimation Method
 - ❖ Relies on the known Attached Synchronization Markers (ASMs) at the start of every frame
 - ❖ Simple implementation
 - ❖ Necessary to have several frames to make an estimation
 - ❖ Not ideal in for a fast changing channel
 - Method 2: The Blind Estimation Method
 - ❖ Makes an estimate with 1 frame of normalized unknown data
 - ❖ Complicated and iterative process to arrive at the estimation
 - ❖ Use of an entire frame not ideal for fast changing channels
 - Method 3: Simulation & Look-Up Table Method
 - ❖ Makes an estimate with 1/2 frame of unknown data
 - ❖ Requires additional gain control to hold the received signal near a 512
 - ❖ Can be used for a faster changing channel



Simulink Block for Pilot-Guided Estimation Method



- Based on the Matlab code used in the analysis of the Pilot-Guided Estimation Method (Method 1)
- The mean and variance of the estimator corresponds to the results obtained in analysis.
- Initialization inputs are set in the Matlab workspace
- The outputs were verified against analysis



The Pilot-Guided Estimation Method

- The Pilot-Guided Method uses a known marker, therefore the Probability Density function of a received signal in AWGN (y) is :

$$P(y | x) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2} \sum_{i=1}^N (y_i x - A)^2 \right]$$

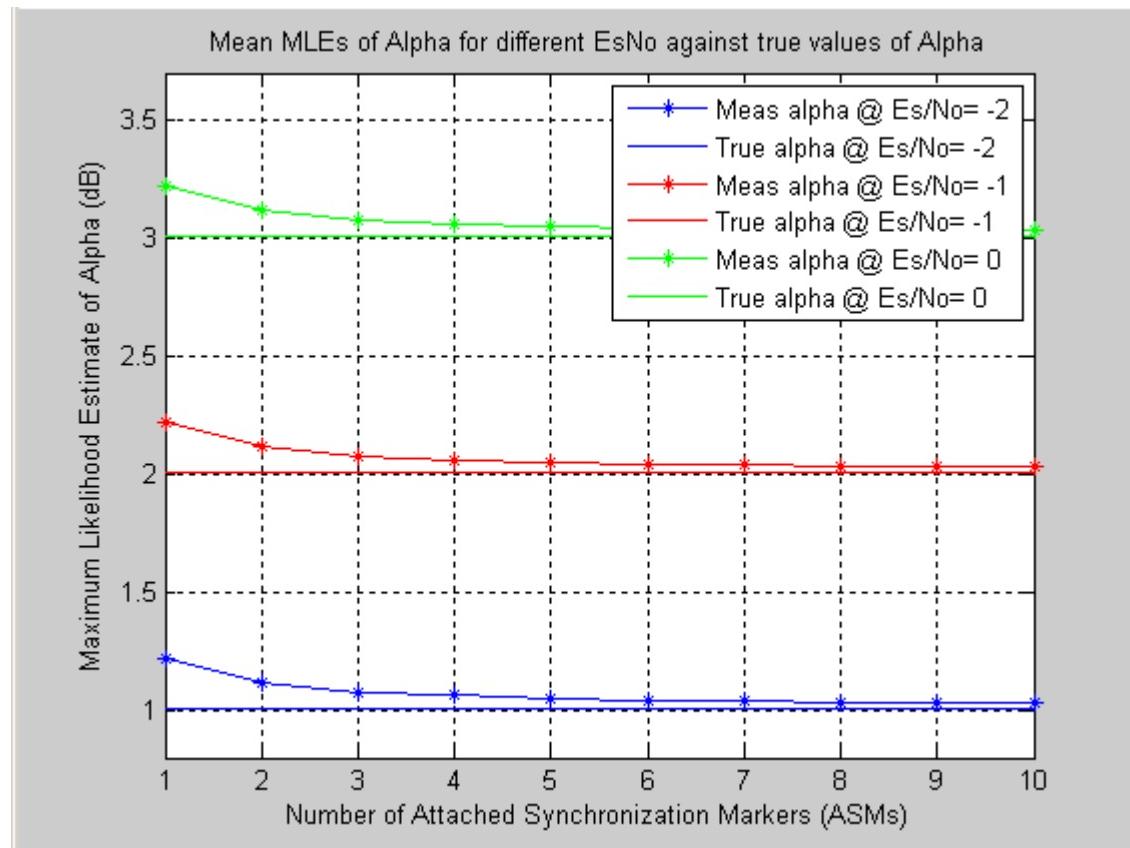
- The derived Maximum Likelihood Estimation's (MLE's) from the Log-Likelihood Ratio are:

$$\hat{A} = \frac{1}{N} \sum_{i=1}^N y_i x \quad \hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N (y_i x)^2 - \hat{A}^2$$

- The estimator (α) is based on the formula: $\alpha = \frac{\langle \hat{x}y \rangle_N}{\langle y^2 \rangle_N - (\langle \hat{x}y \rangle_N)^2}$ where $\langle \cdot \rangle_N$ denotes mean over N symbols
- \hat{x} is usually the detected signal, however since the ASMs are known this is substituted with x
- Analysis will vary the number of ASMs to estimate α at various Es/No
 - 1 ASM to 10 ASM by 1 ASM step
 - Es/No varies from -3 dB to 5 dB in 1 dB increments
- Determine the necessary # of ASM to meet the requirement
 - A Sample Space of 10^5 ASMs was used
 - Area of Interest occurs when Es/No is around -1dB
 - ❖ Operating point of the LDPC decoder to output a BER 10^{-8} occurs when the input BER = 10^{-1}
 - ❖ This occurs when SNR = -0.8 dB



Mean α with Floating Point Output



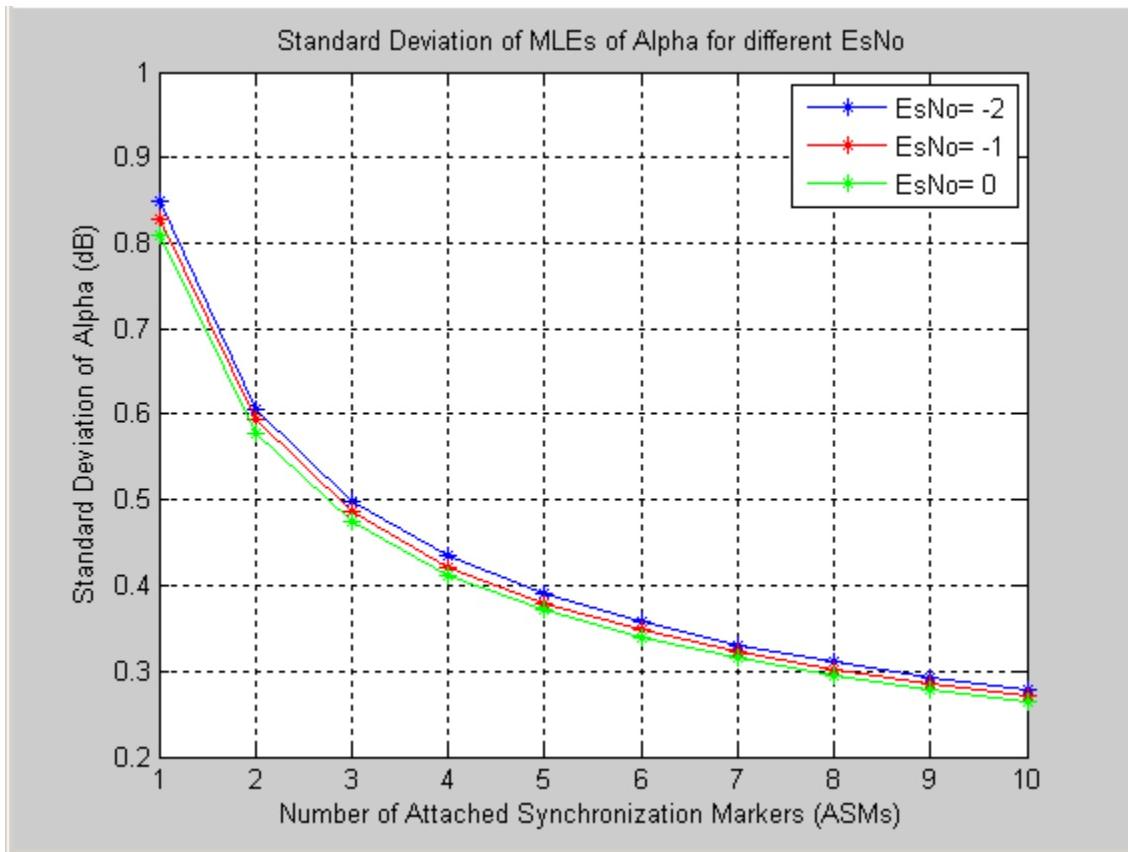
		Es/No (dB)		
		-2	-1	0
1.217522		2.216095	3.217053	1
1.114157		2.115808	3.115565	2
1.077571		2.079883	3.077172	3
1.064476		2.062596	3.061879	4
1.049236		2.049791	3.051148	5
1.044029		2.045864	3.043752	6
1.039201		2.039006	3.041223	7
1.035527		2.036974	3.034679	8
1.033928		2.03212	3.031851	9
1.029219		2.030972	3.030898	10
1.0103	True Values	2.0103	3.0103	

of AS Markers



Standard Deviation of α with Floating Point Output

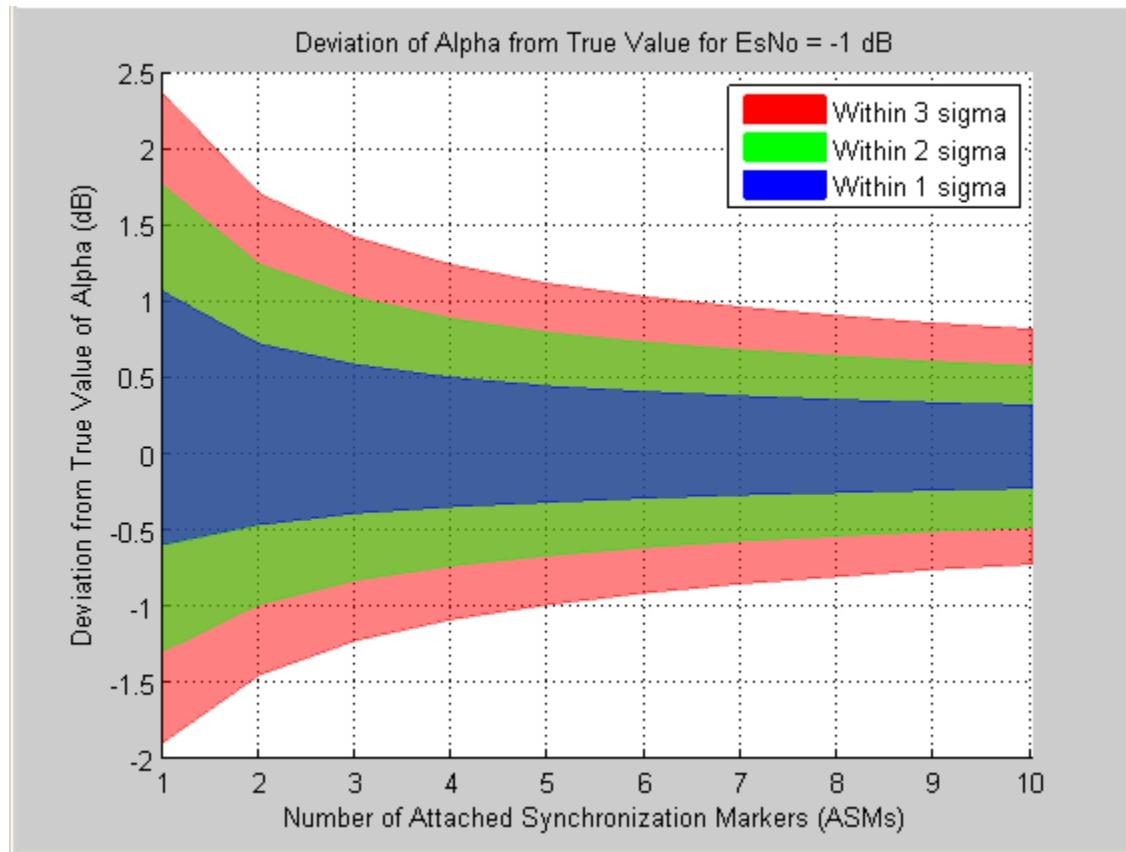
eSCG
ENGINEERING & SCIENCE CONTRACT GROUP



# of AS Markers	Es/No (dB)			
	-2	-1	0	
1	0.848935	0.829203	0.808727	1
2	0.605958	0.593092	0.577789	2
3	0.497243	0.487595	0.474262	3
4	0.435476	0.421558	0.412235	4
5	0.391774	0.379119	0.371318	5
6	0.358225	0.347643	0.338692	6
7	0.330791	0.322965	0.315245	7
8	0.310266	0.302324	0.294575	8
9	0.292963	0.284797	0.277866	9
10	0.278822	0.271027	0.26396	10



Deviation of Combining Ratio Using the Pilot-Guided Estimation Method





Summary of the Pilot-Guided Estimation Method



- It is guaranteed that all estimates within 1 standard deviation of the mean estimate (68%) will meet the requirement of being within 0.5dB of the true value
- Some estimates within 2 standard deviations (95%) may NOT meet this requirement



The Blind Estimation Method

- Assumes that there is equal probability (in a BPSK scheme), therefore the Probability Density function of a received signal is:

$$P(y) = \prod_{i=1}^N \frac{1}{2} \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i - A)^2 / 2\sigma^2} + \frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i + A)^2 / 2\sigma^2} \right]$$

- The Blind Estimation Method uses the Log-Likelihood Ratio to find an MLE for the signal amplitude, which is:

$$\hat{A} = \frac{1}{N} \sum_{i=1}^N y_i \tanh\left(\frac{Ay_i}{\sigma^2}\right)$$

- Using a single frame, step through an algorithm to estimate
 - The iterative algorithm is used to find a numeric approximation \hat{A}
 - Noise power is calculated once a signal amplitude is determined
- Determine the necessary iterations of the algorithm to meet requirements
 - A sample space of 10^5 frames was used
 - Area of Interest occurs when Es/No is around -1dB
 - ❖ Operating point of the LDPC decoder to output a BER 10^{-8} occurs when the input BER = 10^{-1}
 - ❖ This occurs when SNR = -0.8 dB



The Blind Estimation Method Algorithm

$$A = \frac{1}{N} \sum_{i=1}^N y_i \tanh\left(\frac{Ay_i}{\sigma^2}\right)$$

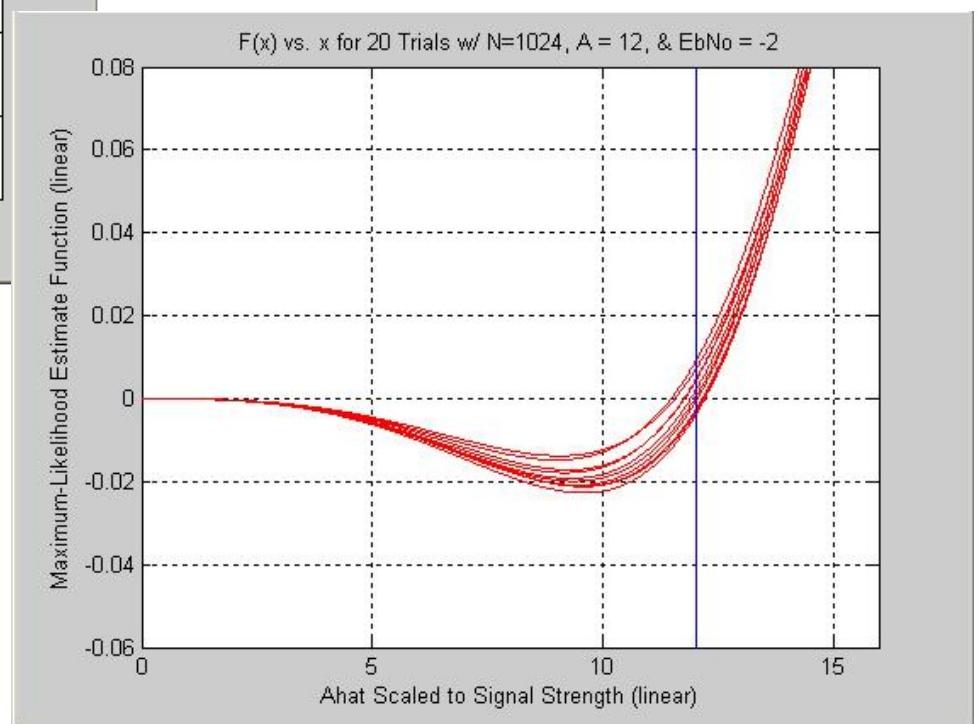
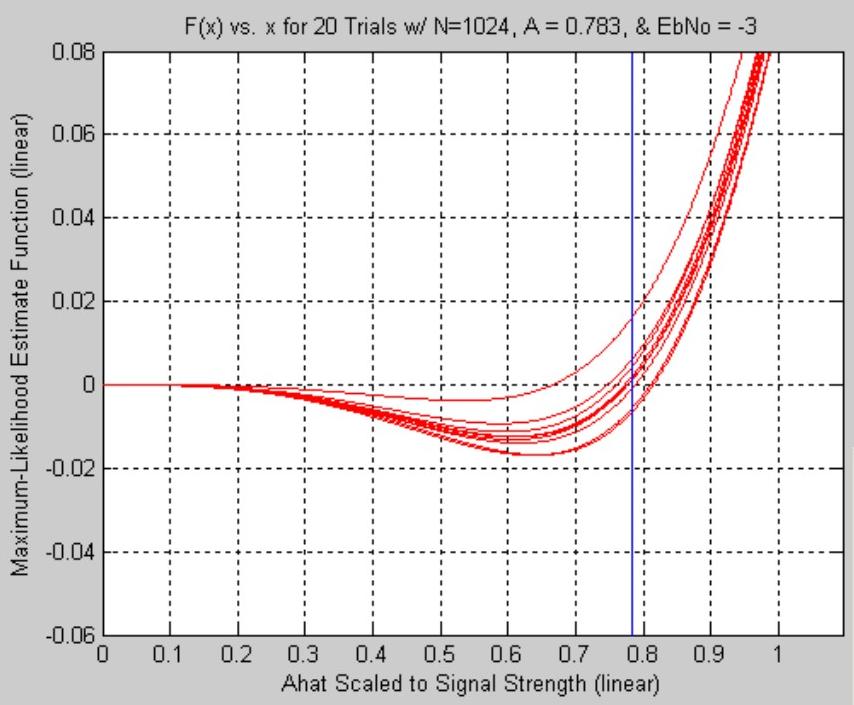
Where:

- ❖ A is the estimate of the input signal amplitude
- ❖ y_i is the amplitude of the received bit
- ❖ σ^2 is the noise power
- ❖ N is the number of bits in the frame

- **1) Normalize the received frame of bits such that the average power of a received bit is 1**
 - This allows noise power to be defined: $\sigma^2 = 1 - A_{mid}^2$
 - Limits the range of estimated signal amplitude to be between 0 and 1
- **2) Set A_{min} near 0 and $A_{max} = 1 - A_{min}$.**
 Note that A_{min} cannot be 0 as this generates the trivial solution A = 0.
- **3) Calculate the midpoint of the expected range and set this as the estimated signal amplitude.**
- **4) Calculate noise power from signal amplitude.**
- **5) Evaluate the equation above with these calculated values.**
- **6) If step 5 is greater than 0 set A_{max} to the midpoint, otherwise set A_{min} to the midpoint.**
- **7) Return to step 3 for the proscribed number of iterations.**
- **8) Take the midpoint one final time and set that as the expected signal amplitude (A_{mid}). Use this value to calculate:** $\alpha = \frac{A_{mid}}{1 - A_{mid}^2}$

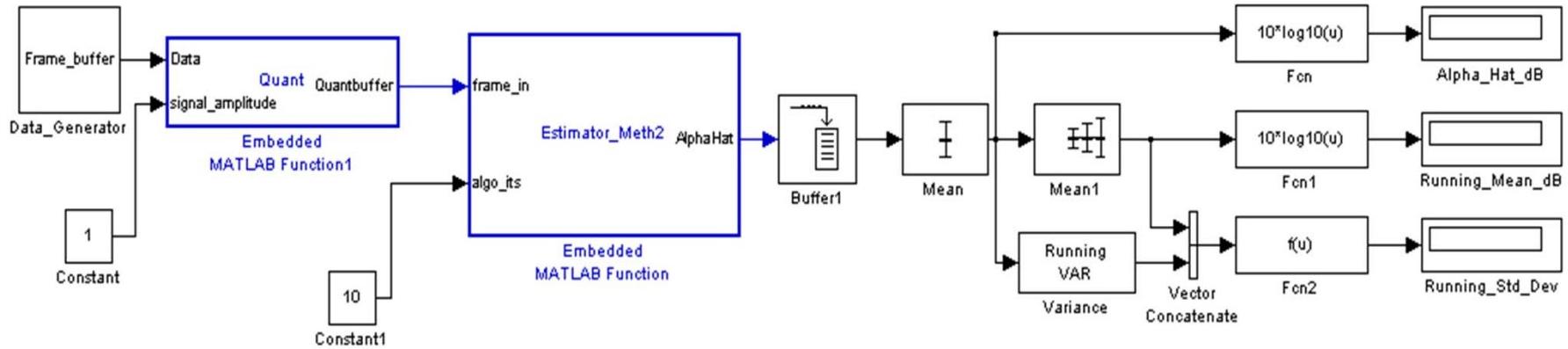


Method 2: Why Step 6?





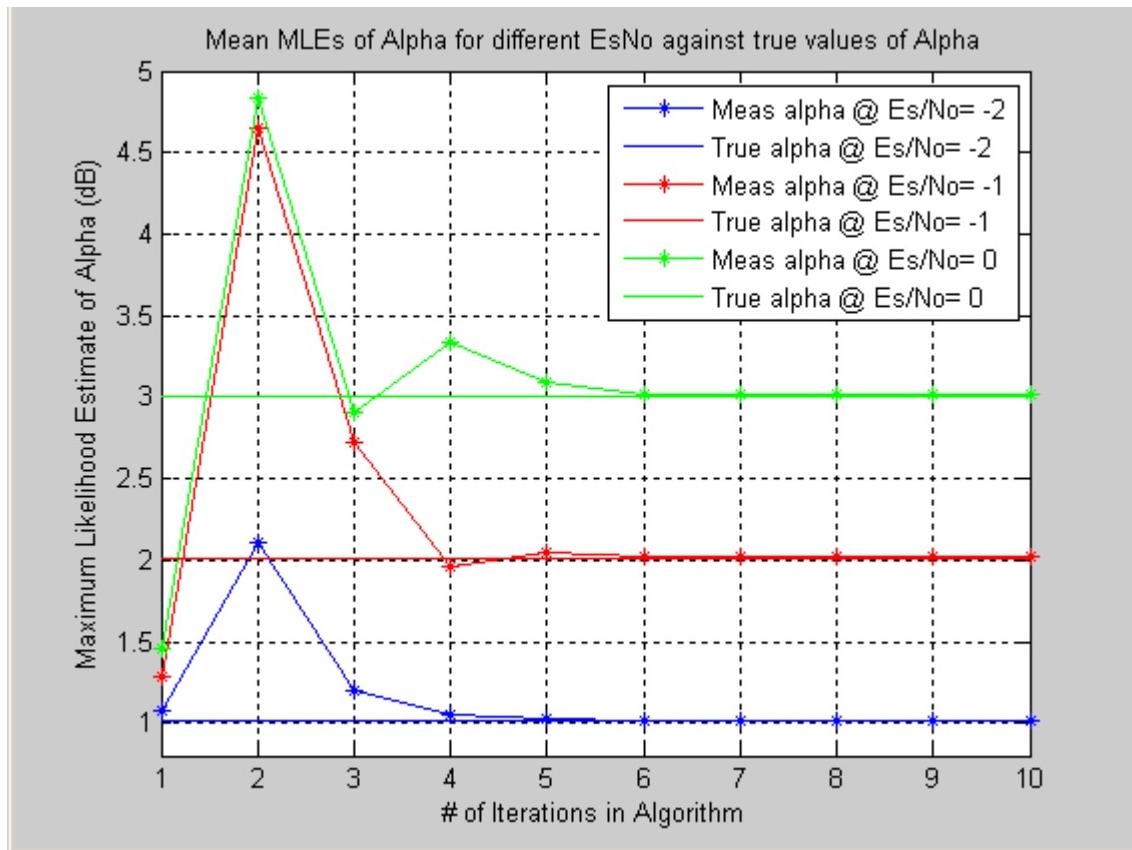
Simulink Blocks for Blind Estimation Method



- Based on the Matlab code used in the analysis of the Blind Method (Method 2)
- The mean and variance of the estimator corresponds to the results obtained in analysis.
- Initialization inputs are set in a Matlab workspace.
- The outputs were verified against analysis



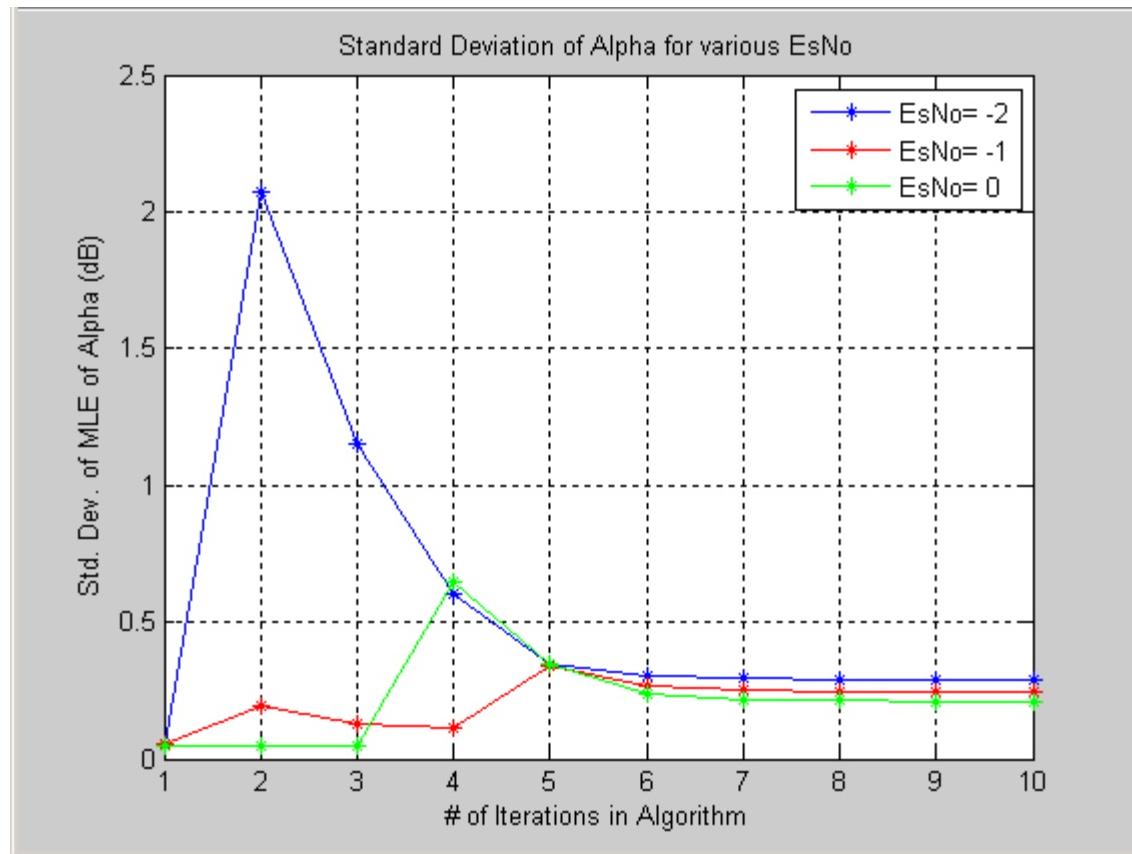
Mean α with Floating Point Output



Es/No (dB)	-2	-1	0	# of Iterations through the algorithm
1.07344	1.07344	1.280814	1.4603	1
2.111958	2.111958	4.647363	4.8386	2
1.203159	1.203159	2.71742	2.9043	3
1.055411	1.055411	1.959882	3.3382	4
1.026691	1.026691	2.04484	3.0899	5
1.018541	1.018541	2.019113	3.02	6
1.017315	1.017315	2.017903	3.0173	7
1.016933	1.016933	2.017271	3.0169	8
1.016824	1.016824	2.017169	3.0167	9
1.016876	1.016876	2.017156	3.0168	10
1.0103	1.0103	2.0103	3.0103	True Values



Standard Deviation of α with Floating Point Output



	Es/No (dB)			# of Iterations through the algorithm
	-2	-1	0	
0.055095	0.052422	0.049486	1	1
2.075299	0.194406	0.049486	2	2
1.147862	0.128529	0.049486	3	3
0.602455	0.114091	0.645361	4	4
0.34819	0.337665	0.345355	5	5
0.303567	0.26505	0.238655	6	6
0.293566	0.249758	0.218892	7	7
0.291016	0.24579	0.21299	8	8
0.290417	0.244789	0.211533	9	9
0.290258	0.244584	0.211125	10	10



Deviation of Combining Ratio Using the Blind Estimation Method





Summary of the Blind Estimation Method

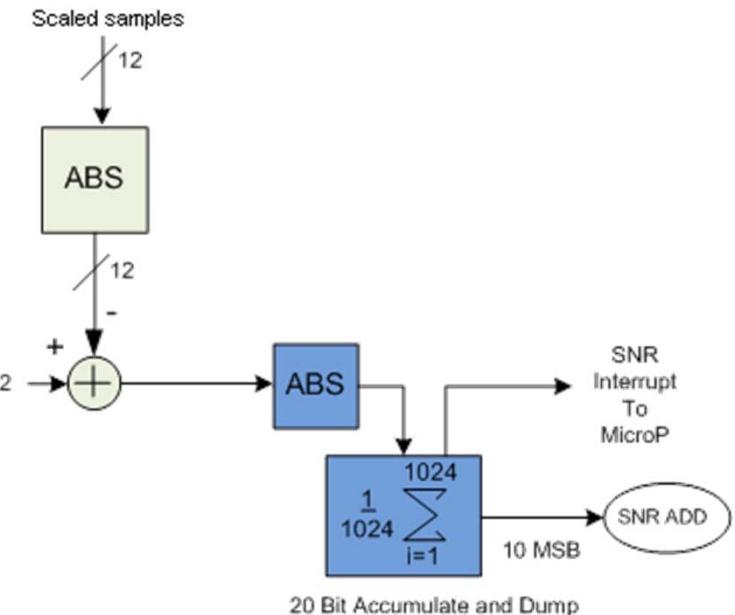


- It is guaranteed that all estimates within 1 standard deviation of the mean estimate (68%) will meet the requirement of being within 0.5 dB of the true value
- At least 9 iterations of the binary search are needed before additional iterations are negligible as compared to the variance of the data



Prior Simulation – Look-up Table Method

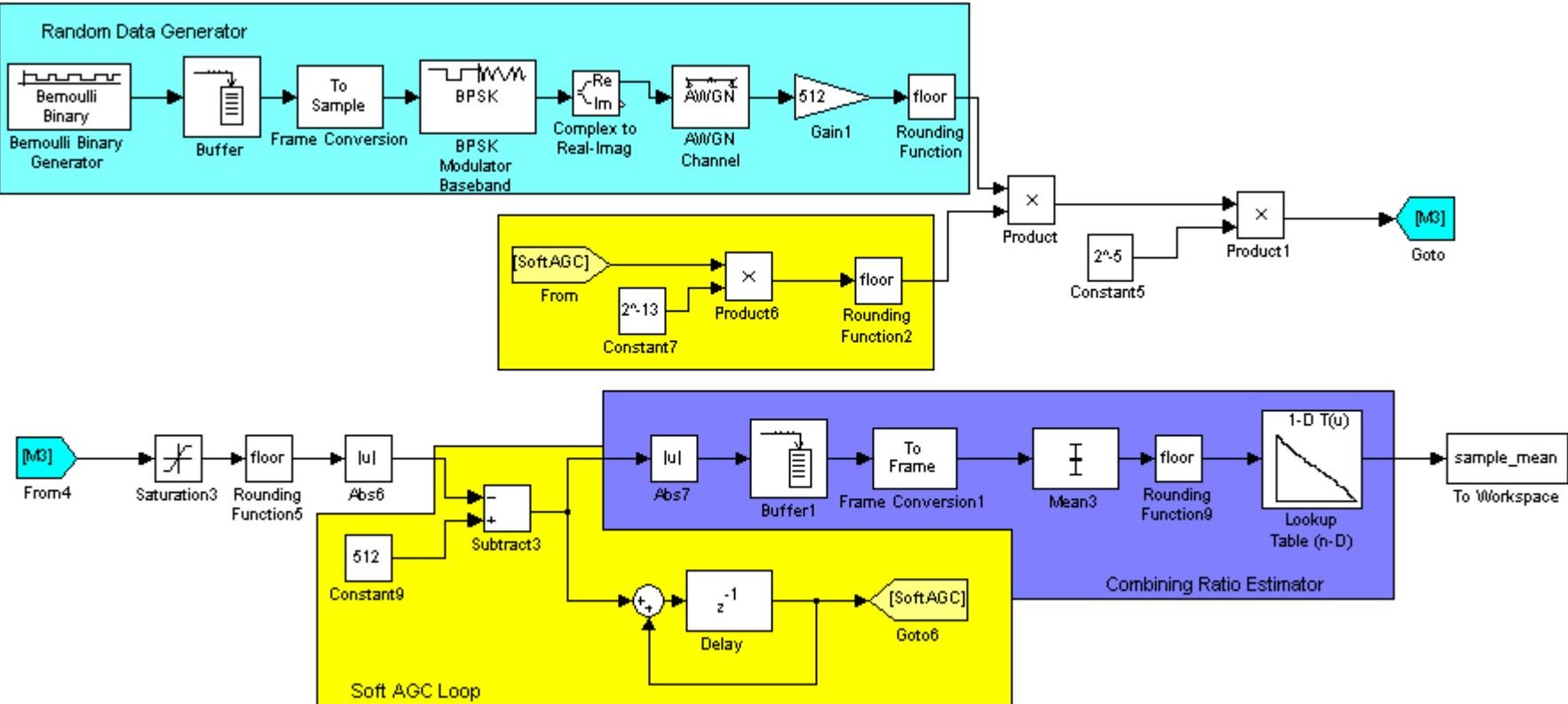
- Uses prior simulation to determine:
$$k * E\left\{1 - \frac{|y|}{\mu_{|y|}}\right\}$$
- For each SNR the expected value is determined by simulation and placed in the look-up table
- The deviation around the specified mean, k, is averaged over the same number of samples
- The average (SNR ADD in the diagram) is sent to the look-up table to determine the combining ratio
- It is important to control the strength of the received signal in order to accurately estimate the signal deviation.





MATLAB Simulation: Combining Ratio Estimate

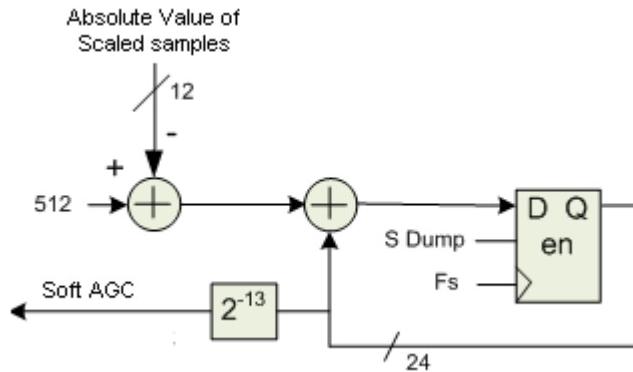
es²G
ENGINEERING & SCIENCE CONTRACT GROUP



- In the AWGN channel block Es/No was applied from -1.5 dB – 8.5 dB in 0.5 dB steps
- Statistics were calculated over 100000 estimates of the Combining Ratio, estimated from 1024 soft decision bits.



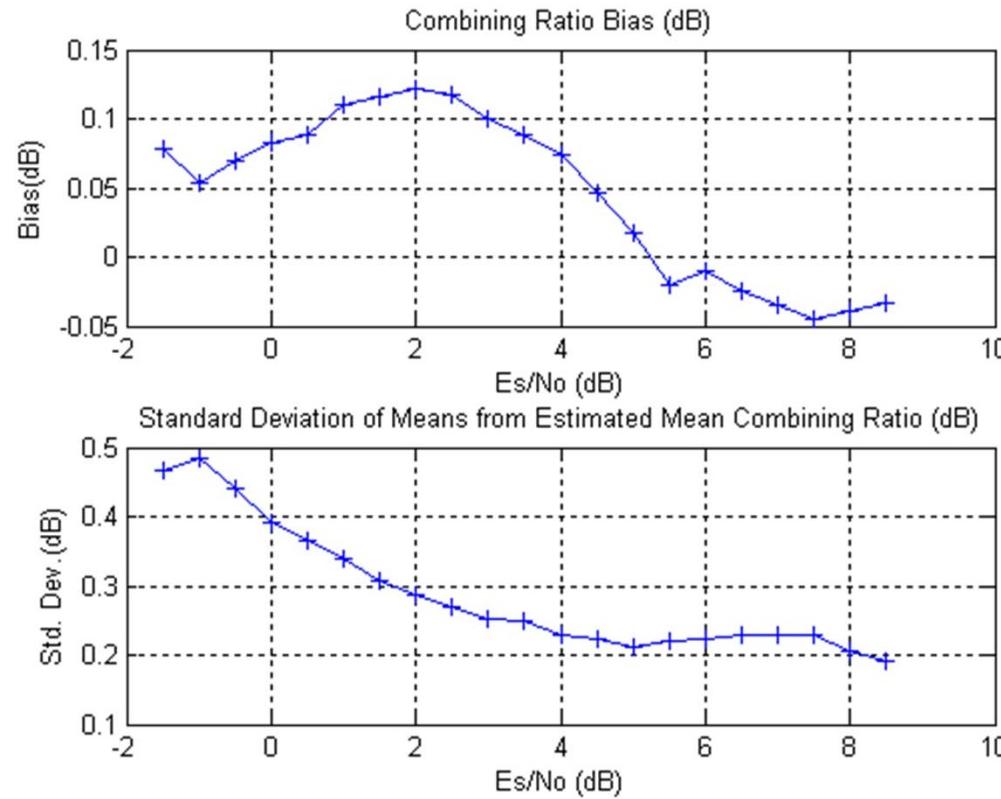
Automatic Gain Control



- Provides feedback to keep the received signal within the range of the estimation
- This will adjust the gain of the input samples based on the deviation from the baseline 512 point
- The deviation is accumulated and the 11 most significant bits is the 'Soft AGC' factor



Results

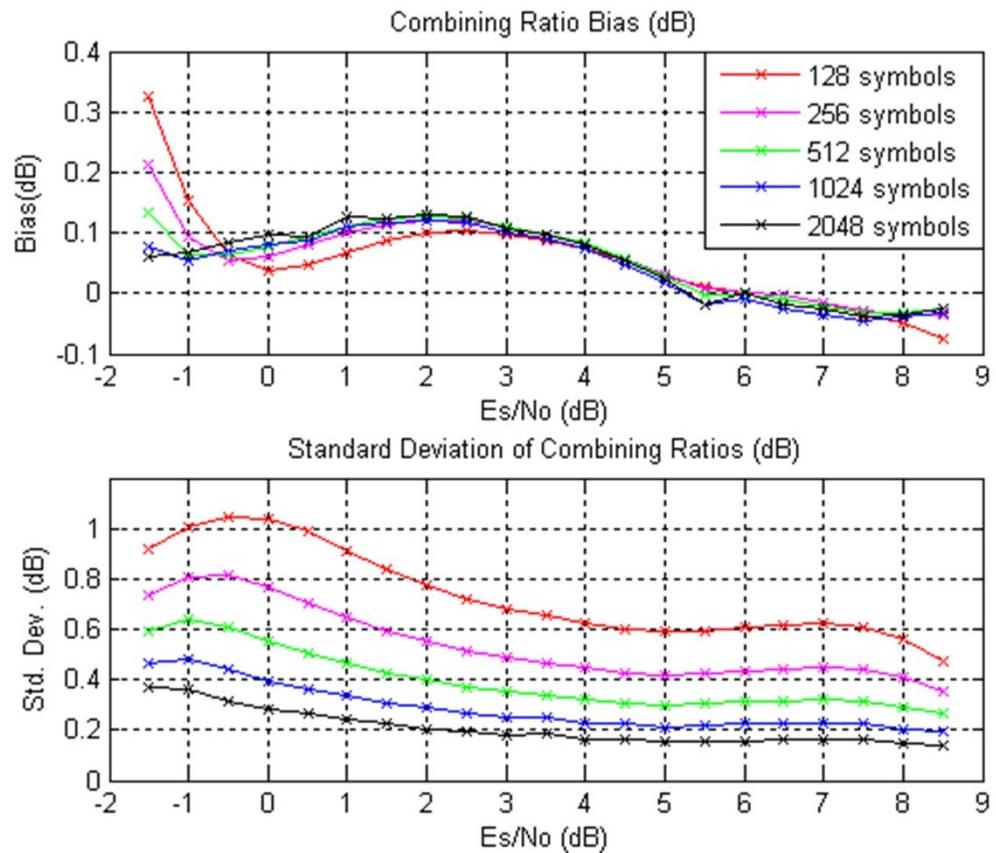


- **Bias is within 0.15 dB**
- **Standard Deviation (STD) is within 0.5 dB**



Effects of the Number of Symbols used to Calculate the Combining Ratio

- Bias is close to unchanged
- The STD decreases with an increase in the number of symbols used for the Combining Ratio estimation
- From 128 symbols to 256 symbols = 0.25dB improvement in the STD
- From 1024 symbols to 2048 symbols there is a 0.1dB improvement in the STD





Summary of Prior Simulation – Look-up Table Method



- **The Combining Ratio Estimate is within 0.5 dB of its true value for 71.2% of the data at Es/No = -1 dB (Operating Point)**
 - The Combining Ratio Estimate is within 0.5 dB of its true value more than for 98% of the data at Es/No = 8.5



Summary

- The three methods for estimating the Combining Ratio – the Pilot-Guided Estimation Method, the Blind Estimation Method , and a Prior Simulation – Look-up Table Method – are within 0.5 dB for one STD of the CR at $Es/No = -1$ dB.
- The Pilot-Guided Estimation Method was easy to implement; however, 6 frames of data were required to estimate one STD of the CR within 0.5 dB of the true CR.
- The Blind Estimation Method was complicated to implement and one frame of data estimated one STD of the CR within 0.5 dB of the true CR.
- The Prior Simulation – Look-up Table Method using 1024 sample estimated one STD of the CR within 0.5 dB of the true CR.



References

- Andrews, Kenneth, et al. “Symbol Scaling for LDPC Decoders.” February 6, 2008.
- Li, Bin, et al. “A Low Bias Algorithm to Estimate Negative SNRs on an AWGN Channel.” IEEE Communications Letters, vol. 6, no.11, November 2002.
- Gagliardi, Robert M., Thomas, C. Melvil. “PCM Data Reliability Monitoring Through Estimation of Signal-to-Noise Ratio.” IEEE Transactions on Communication Technology, vol. com-16, no.3, June 1968.
- Performance Evaluation and Modeling of ‘SNR’ Estimators (EV7-09-4473)
- Modeling and Performance Evaluation of the Combining Ratio Estimator in the Orion S-Band Transponder (EV6-12-4732)
- Combining Ratio Table Lookup for LDPC Decoder (Tech Memo 12192-TM026)
- Soft Decision Processing (Tech Memo 12192-TM022)
- Symbol Scaling for LDPC Decoders: Kenneth Andrews, Sam Dolinar, Dariush Divsalar, Jon Hamkins, Vic Vilnrotter



Extra Slides





Maximum-Likelihood Estimates (MLEs)



- The MLE maximizes either Likelihood Ratio or the Log-Likelihood Ratio
- MLE of a ratio is equal to the ratio of MLEs $\hat{\alpha} = \frac{\hat{A}}{\hat{\sigma}^2}$
- Method 1 uses a known marker, therefore the Probability Density function of a received signal (y) is :

$$P(y | x) \Rightarrow \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[\frac{-1}{2\sigma^2} \sum_{i=1}^N (y_i x - A)^2 \right]$$

- Method 2 assumes that there is equal probability (in a BPSK coding scheme), therefore the Probability Density function of a received signal is:

$$P(y) = \prod_{i=1}^N \frac{1}{2} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-(y_i - A)^2/2\sigma^2} + \frac{1}{\sigma\sqrt{2\pi}} e^{-(y_i + A)^2/2\sigma^2} \right]$$

- We find the maximum by setting the derivative to 0



Determining Log-Likelihood Ratio for the Pilot-Guided Method

$$\frac{\partial}{\partial A} \ln \left(\frac{1}{(2\pi\sigma^2)^{N/2}} \exp \left[\frac{-1}{2\sigma^2} \sum_{i=1}^N (y_i x - A)^2 \right] \right) = \frac{\partial}{\partial A} \left(\ln \left(\frac{1}{(2\pi\sigma^2)^{N/2}} \right) + \left[\frac{-1}{2\sigma^2} \sum_{i=1}^N (y_i x - A)^2 \right] \right)$$

$$0 = \frac{1}{\sigma^2} \sum_{i=1}^N (y_i x - A)$$

$$0 = \frac{1}{\sigma^2} \sum_{i=1}^N y_i x - \frac{N}{\sigma^2} A$$

$$\hat{A} = \frac{1}{N} \sum_{i=1}^N y_i x$$

$$\frac{\partial}{\partial \sigma^2} \ln \left(\frac{1}{(2\pi\sigma^2)^{N/2}} \exp \left[\frac{-1}{2\sigma^2} \sum_{i=1}^N (y_i x - A)^2 \right] \right) = \frac{\partial}{\partial \sigma^2} \left(\ln \left(\frac{1}{(2\pi\sigma^2)^{N/2}} \right) + \left[\frac{-1}{2\sigma^2} \sum_{i=1}^N (y_i x - A)^2 \right] \right)$$

$$0 = \frac{-N}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^N (y_i x - A)^2$$

$$\frac{N}{2\sigma^2} = \frac{1}{2\sigma^4} \sum_{i=1}^N (y_i x - \hat{A})^2$$

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N (y_i x - \hat{A})^2$$

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N (y_i x)^2 - 2 * \frac{1}{N} \sum_{i=1}^N \hat{A}(y_i x) + \hat{A}^2$$

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N (y_i x)^2 - \hat{A}^2$$



Determining Log Likelihood Equation for the Blind Method pt. 1

esCG
ENGINEERING & SCIENCE CONTRACT GROUP

$$\frac{\partial}{\partial A} \ln P(y) = 0 \Rightarrow 0 = \frac{\partial}{\partial A} \ln \prod_{i=1}^N \left[\frac{1}{2} \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i - A)^2 / 2\sigma^2} + \frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i + A)^2 / 2\sigma^2} \right] \right]$$

$$0 = \frac{\partial}{\partial A} \ln \prod_{i=1}^N \left[\frac{1}{2} \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i^2 - 2Ay_i + A^2) / 2\sigma^2} + \frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i^2 + 2Ay_i + A^2) / 2\sigma^2} \right] \right]$$

$$0 = \frac{\partial}{\partial A} \ln \prod_{i=1}^N \left[\frac{1}{2} \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i^2 + A^2) / 2\sigma^2} e^{2Ay_i / 2\sigma^2} + \frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i^2 + A^2) / 2\sigma^2} e^{-2Ay_i / 2\sigma^2} \right] \right]$$

$$0 = \frac{\partial}{\partial A} \ln \prod_{i=1}^N \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i^2 + A^2) / 2\sigma^2} \left(\frac{e^{2Ay_i / 2\sigma^2} + e^{-2Ay_i / 2\sigma^2}}{2} \right) \right]$$

$$0 = \frac{\partial}{\partial A} \ln \prod_{i=1}^N \left[\frac{1}{\sigma \sqrt{2\pi}} e^{-(y_i^2 + A^2) / 2\sigma^2} \cosh\left(\frac{Ay_i}{\sigma^2}\right) \right]$$



Determining Log Likelihood Equation for the Blind Method pt. 2

$$0 = \frac{\partial}{\partial A} \ln \left(\frac{1}{2\pi\sigma^2} \right)^{N/2} + \frac{\partial}{\partial A} \ln e^{\left(\frac{-1}{2\sigma^2} \right) \sum_{i=1}^N y_i^2 + A^2} + \frac{\partial}{\partial A} \ln \prod_{i=1}^N \cosh \left(\frac{Ay_i}{\sigma^2} \right)$$

$$0 = \frac{\partial}{\partial A} \left(\frac{-1}{2\sigma^2} \right) \sum_{i=1}^N (y_i^2 + A^2) + \frac{\partial}{\partial A} \sum_{i=1}^N \ln \left(\cosh \left(\frac{Ay_i}{\sigma^2} \right) \right)$$

$$0 = \frac{-N}{\sigma^2} A + \sum_{i=1}^N \frac{1}{\cosh \left(\frac{Ay_i}{\sigma^2} \right)} \sinh \left(\frac{Ay_i}{\sigma^2} \right) \frac{y_i}{\sigma^2}$$

$$0 = \frac{-N}{\sigma^2} A + \frac{1}{\sigma^2} \sum_{i=1}^N y_i \tanh \left(\frac{Ay_i}{\sigma^2} \right)$$

$$0 = \frac{-N}{\sigma^2} \left(A - \frac{1}{N} \sum_{i=1}^N y_i \tanh \left(\frac{Ay_i}{\sigma^2} \right) \right)$$

$$0 = A - \frac{1}{N} \sum_{i=1}^N y_i \tanh \left(\frac{Ay_i}{\sigma^2} \right)$$

$$\hat{A} = \frac{1}{N} \sum_{i=1}^N y_i \tanh \left(\frac{Ay_i}{\sigma^2} \right)$$